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Propagation of current waves along a transmission line with randomly located non-uniformities inside a rectangular resonator

Sergey V. Tkachenko[1], **Jürgen B. Nitsch**[1], **Moustafa Raya**[1], **Ronald Rambousky**[2], **and Ralf Vick**[1]

[1]Otto-von-Guericke University Magdeburg, Magdeburg 39106, Germany
[2]Bundeswehr Research Institute for Protective Technologies and NBC Protection (WIS), Munster, Germany

Correspondence: Sergey V. Tkachenko (sergey.tkachenko@ovgu.de)

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Abstract. In this paper, we investigate the propagation of high-frequency current waves along a stochastic transmission line inside a rectangular cavity using as basis the model of a transmission line with the symmetry of the resonator. The stochastization of the line is created by a stochastically arranged chain of loads. Using similar models one also can take into account stochastic geometry of the line. Research has shown a significant difference between the propagation of the current wave along the transmission line with stochastic loads in free space and in the resonator. In the first case, the average square of the absolute value of the transmission coefficient exponentially tends to zero with increasing length of the line due to interference phenomena for current waves. In the second case, in the average, the current can penetrate through the stochastic chain of the loads due to the re-reflection of the signal from the walls of the resonator.

1 Introduction

Investigation of electromagnetic coupling to antennas and transmission lines inside resonators – like objects as shielded rooms, computer cases, aircraft fuselages, satellites, etc., is a challenging task in modern electromagnetic compatibility. The development of corresponding calculation methods is a quite advanced mathematical problem. In reality the problem is even more complicated, because the exact geometrical and electrical parameters of the transmission line, which define the coupling, are unknown in practical applications. As result one can only talk about the probability of one or the other parameters.

Numerical methods – like MoM, TLM, etc. – are usually applied for the solution of this group of problems. But they are applicable for specific cases only and are computationally very intensive. In contrast to this, analytical and semi-analytical methods are applicable on general cases, and allow to make a fast analysis of the problem.

In this paper, we investigate the propagation of high-frequency current waves along a stochastic transmission line inside a rectangular cavity. Inside the resonator the wire is conducted parallely to four walls of the resonator and connects two other opposite walls. This system allows an exact analytical solution by a spatial Fourier transformation for any kind of excitation, including any finite number of lumped sources and loads, which can be considered as controlled voltage sources (Tkachenko et al., 2013a). In the case of multiple loads the problem is reduced to the solution of a linear algebraic system of Nth order (N is the total number of sources and loads) (Tkachenko et al., 2014a). This method is computationally effective, fast – it contains only double mode sums- and allows to solve stochastic problems with relative large statistical samples.

In the present research, the stochastization of the line is created by the chain of randomly arranged lumped impedances, which can have reactive as well ohmic components. Such model was earlier investigated for the case of free space (Tkachenko et al., 2013b). It was shown that current waves cannot propagate through such chain of the loads due to the wave interference effects. If the randomization of the scatterers positions is large enough, the average square of the absolute value of transmission coefficient D decreases (exponentially) with increasing length of the chain or (and) increasing parameter of stochastization. A probability density function (PDF) of this value also decreases exponentially with increasing $|D|^2$. Moreover, it was shown that – with proper choice of parameters – this model describes lines with

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stochastic geometry (Tkachenko et al., 2016; Vick, 2014) in the sense that the mean values of $|D|^2$ and its PDF behave in a similar way with frequency. The present research has shown a significant difference between the propagation of the current waves along the transmission line with stochastically arranged loads in free space and inside a resonator. In the resonator the current can penetrate through the stochastical chain of the loads due to the re-reflection of the signal from the walls. This means that the averaged square of the absolute value of the transmission coefficient does not decrease with increasing parameter of randomization of the chain, and correspondingly the PDF has non-zero "tail" for a $|D|^2$ value of about one.

This paper is organized as follows: In Sect. 2 we shortly describe the solution method for the transmission line with symmetrical geometry and arbitrary number of lumped loads obtained in earlier papers (Tkachenko et al., 2011, 2013a, 2014a). In Sect. 3 we investigate the statistical moments of the transmission coefficient for the current wave through the transmission line with a randomly arranged set of mainly inductive lumped loads. This section is concluded by a comparison of the results for the cases of the transmission line in free space and the transmission line in the resonator. In Sect. 4 we investigate the PDF for the square of the absolute value of the transmission coefficient and will offer an analytical model to describe this PDF. Section 5 concludes the paper.

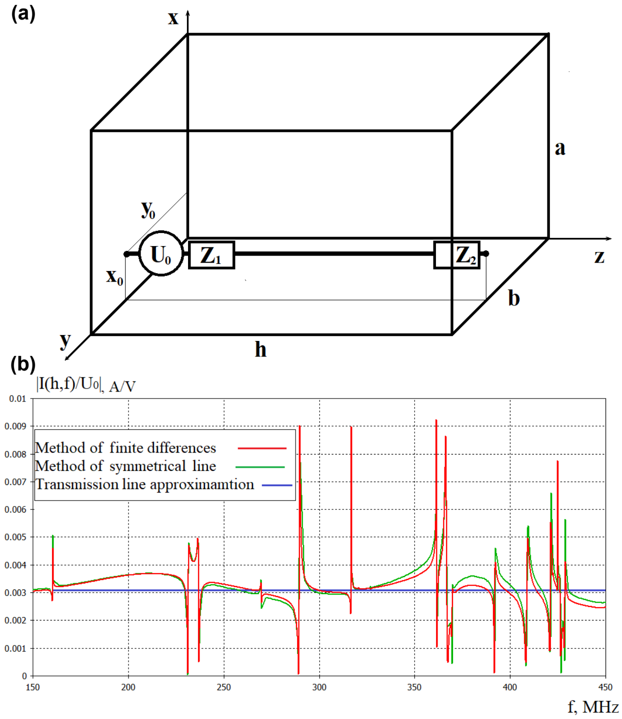

Figure 1. The normalized current along the symmetrical TL with the following geometry. Parameters of resonator: $a = 1.5$ m, $b = 1.2$ m, $h = 0.9$ m. Parameters of transmission line: $L = h = 0.9$ m, $h_{\mathrm{TL}} = x_0 = 9$ cm, $y_0 = 37$ cm, $r_0 = 1$ mm, $Z_1 = 0$, $Z_2 = Z_C = 311\,\Omega$.

2 Method of Symmetrical Lines inside Resonators (MoSL)

2.1 Exact solution for the short circuited wire

First we consider a short-circuited transmission line with symmetrical geometry inside the resonator with the side lenghts a, b and h. The line is parallel to one axis of the resonator and connects two opposite walls (see Fig. 1). Consider an electrical field $\boldsymbol{E}^{\mathrm{ex}}(\boldsymbol{r})$ inside the box excited, e.g., by a radiating antenna, a penetration of external electromagnetic waves through slots and apertures or by a lumped voltage source. This field excites in the wire an electrical current $I(\omega, z)$, which, in turn, is the source of the scattered field $\boldsymbol{E}^{\mathrm{sc}}(\boldsymbol{r})$, which is calculated by integration with the tensor (dyad) Green's function of the resonator $\overline{\overline{G}}^{\mathrm{E}}(\boldsymbol{r}, \boldsymbol{r}_1)$. For the perfectly conducting thin wire the tangential component of the total (exciting plus scattered) electrical field on the boundary of the wire equals zero, yielding the Electrical Field Integral Equation (EFIE) for the induced current:

$$\int_0^L \overline{\overline{G}}{}^{\mathrm{E}}_{zz}(x_0 + r_0, y_0, z, x_0, y_0, z_1)\; I(z_1)\mathrm{d}z_1 + E_z^{\mathrm{ex}}(z) = 0 \quad (1)$$

Here $\overline{\overline{G}}{}^{\mathrm{E}}_{zz}$ is the zz component of the tensor Green's function (electrical current $\rightarrow$ electrical field) for the rectangular resonator, x_0, y_0 are the positions of the wire in the x, y plane, r_0 is the radius of the wire.

The quantity $\overline{\overline{G}}{}^{\mathrm{E}}_{zz}$ is given by:

$$G^{\mathrm{E}}_{zz}(\boldsymbol{r}, \boldsymbol{r}_1) = \frac{4\eta_0}{jkV} \sum_{\substack{m_1 = 1\\ m_2 = 1\\ m_3 = 0}}^{\infty} \frac{\varepsilon_{m_3}(k^2 - (k_z^v)^2)\sin(k_x^v x_1)\sin(k_x^v x)\sin(k_y^v y_1)\sin(k_y^v y)\cos(k_z^v z_1)\cos(k_z^v z)}{k_v^2 - k^2 + j\delta} \quad (2)$$

with the following definitions: $V = abh$ is the volume of the resonator, $k_x^v = m_1\pi/a$, $k_y^v = m_2\pi/b$, $k_z^v = m_3\pi/h$, $k^v = \sqrt{(k_x^v)^2 + (k_y^v)^2 + (k_z^v)^2}$, $\eta_0 = \sqrt{\mu_0/\varepsilon_0}$, $\varepsilon_m = 2 - \delta_{m,0}$.

For the short-circuited wire ($Z_1 = Z_2 = 0$ in the Fig. 1) the induced current satisfies the zero Neumann boundary condition:

$$\partial I(z)/\partial z|_{z=0} = \partial I(z)/\partial z|_{z=h} = 0 \quad (3)$$

The EFIE for symmetrical thin-wires has the same symmetry as an electrodynamic problem for the empty chamber: translational symmetry with translational constant h. A corresponding representation of the translation group is given by the collection of cosine-functions $\{\cos(m_3\pi z/h)\}$. We decompose both the exciting field and unknown current with respect to this orthogonal system of functions:

$$E_z^{\mathrm{ex}}(\boldsymbol{r}) = \sum_{m_3=0}^{\infty} E_z^{\mathrm{ex}}(x,y,m_3)\cdot\cos(m_3\pi z/h), \tag{4a}$$

$$I(z) = \sum_{m_3=0}^{\infty} I(m_3)\cdot\cos(m_3\pi z/h) \tag{4b}$$

Then we substitute Eq. (4a, b) into Eq. (1) and take into account Eq. (2). After some transformative calculations, the orthogonality of the cosine functions is used to obtain the Fourier components of the current.

$$I(m_3) = \frac{E_z^{\mathrm{ex}}(x_0,y_0,m_3)\,jk}{\eta_0((k_z^v)^2-k^2)\cdot S} \tag{5}$$

Here the function S characterizes two – dimension scattering. After some manipulations it can be written in the form:

$$S = \frac{1}{a}\sum_{m_1=1}^{\infty}\sin^2(k_x^v x_0)\cdot\left[\frac{2\sinh(\widetilde{\gamma}_v(b-y_0))\sinh(\widetilde{\gamma}_v y_0)}{\widetilde{\gamma}_v\sinh(\widetilde{\gamma}_v a)}-\frac{1}{k_x^v}\right] + \frac{1}{2\pi}\ln\left[\frac{a|\sin(\pi x_0/a)|}{\pi r_0}\right],$$
$$\text{where } \widetilde{\gamma}_v = \sqrt{(k_x^v)^2+(k_z^v)^2-k^2} \tag{6}$$

If the frequency ω is far from cavity resonances: $|\omega-\omega_v|>>\Delta\omega$, where $\Delta\omega$ is the shift of an eigen- frequency, which appears due to the interaction of TL modes with those of the resonator: $\Delta\omega_v \sim \omega_v 4\pi\left[abk_v^2\ln(2x_0/r_0)\right]^{-1}$. In case then the wire is close to one of the walls of the resonator: $r_0 \ll x_0 \ll y_0, a$, one can show that $S\approx S_{\mathrm{TL}}\approx \ln(2x_0/r_0)/2\pi$, and Eqs. (4b) and (5) yield the classical TL-solution for the short-circuited wire obtained by a Fourier expansion series (Tkachenko et. al., 2011, 2013a).

2.2 Lumped excitations and loads

The solutions (Eqs. 4b–5) can be re-written in the integral form with Green's function for a transmission line, as in the transmission line approximation, however, with exact scattering function $S(k,m_3,x_0,y_0)$ (Eq. 6), taking into account the properties of the resonator

$$I(z) = \int_0^h E_z^{\mathrm{ex}}(z_1)Y_{\mathrm{RES}}(z,z_1,k,a,b,h)\mathrm{d}z_1$$
$$Y_{\mathrm{RES}}(z,z_1,k,a,b,h,x_0,y_0) := \frac{jk}{h\eta_0}\sum_{m_3=0}^{\infty}\frac{\varepsilon_{m_3}\cos(\pi m_3 z/h)\cos(\pi m_3 z_1/h)}{((k_z^v)^2-k^2)\cdot S(k,m_3,x_0,y_0)} \tag{7}$$

The Eqs. (4b)–(6) or (6)–(7) represent exact analytical solutions for the current induced in the symmetrical line by arbitrary excitations E_z^{ex}. In particular, the excitation of a lumped source with the amplitude U_0 and coordinate z_0, gives the current:

$$E_z^{\mathrm{ex}}(z) = U_0\cdot\delta(z-z_0) \Rightarrow I(z) = U_0\cdot Y_{\mathrm{RES}}(z,z_0) \tag{8}$$

Similarly, one can also include lumped impedances. The lumped load Z at the point z_0 can be considered as controlled lumped source with unknown amplitude:

$$E_z^Z(x_0,y_0,z) = -Z\cdot I(z_0)\delta(z-z_0). \tag{9}$$

In Eq. (9) $I(z)$ represents the total current which flows through the load. The contribution to the current from this load then is

$$I_Z(z) = -Z\cdot I(z_0)\cdot Y(z,z_0). \tag{10}$$

Thus, dealing with "external" exciting fields and/or fields from controlled sources, one can express the total current by unknown currents through loads. Calculating these currents one obtains a linear system for them (Tkachenko et al., 2014a). For example, for the chain of N lumped impedances excited by a lumped source (see Fig. 4a) the total exciting z-field is

$$E_z^{\mathrm{ex.tot}}(z) = U_0\delta(z-z_0) - \sum_{n=1}^{N} Z_n I(z_n)\delta(z-z_n) \tag{11}$$

where z_0 is the position of the exciting lumped source U_0, z_1, z_2, ... z_N are positions of the lumped impedances Z_1, Z_2, ... Z_N. Then, the total current is a sum of $N+1$ terms

$$I(z) = U_0 Y_{\mathrm{RES}}(z,z_0) - \sum_{n=1}^{N} Z_n I(z_n) Y_{\mathrm{RES}}(z,z_n) \tag{12}$$

The current amplitudes in the points z_n can be found by the solution of the linear equation system:

$$\sum_{n=1}^{N}\left[\delta_{n,n1} + Z_{n1}Y_{\mathrm{RES}}(z_n,z_{n1})\right]\cdot I(z_{n1}) = U_0 Y_{\mathrm{RES}}(z_n,z_0) \tag{13}$$

For the case of two terminal loads Eq. (13) can be solved analytically (Tkachenko et al., 2011). An example for one load is shown in Fig. 1b.

3 Transmission Line with randomly located non-uniformities. Statistical Moments

In this Section, we investigate the propagation of current waves along a multiloaded transmission line with randomly located non-uniformities. Earlier, it is was shown (Tkachenko et al., 2013b) that statistical properties of transmission lines with stochastic geometry could be approximately described by the model of a straight transmission line with stochastic lumped inductance-like loads. Calculations in the used method are greatly facilitated by the fact that,

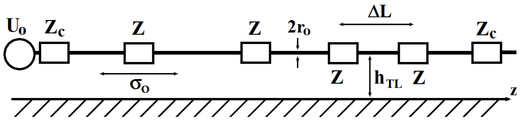

Figure 2. Transmission line with stochastically arranged lumped impedances in free space.

for each Fourier mode the amplitude of the lumped potential and its position define only the nominator of the corresponding Fourier series coefficient through simple trigonometric functions. The denominator is the same for each element, including one-dimensional summation and depending on the transversal geometry of the resonator.

Now, we consider a resonator with the dimensions $a = 30\,\text{cm}$, $b = 53\,\text{cm}$, and $h = 79\,\text{cm}$ (see Fig. 1a). The transversal position of the transmission line is given by $x_0 = a/2$, $y_0 = b/2$. The line is excited by a lumped voltage source $U_0 = 1\,\text{V}$ at the left terminal and loaded by six lumped loads. At the terminals, the loads are matched impedances (for the propagation of TEM waves along the wire in an infinite waveguide): $Z_1 = Z_{\text{Nmax}} = Z_C = 333\,\Omega$. The central part of the line contains $N_{\max} - 2$ equal non-uniformities which can be considered as local distortions of the geometry as well as lumped loads. Here, we assume that they are lumped impedances with an essentially inductive and small ohmic component: $Z_n = Z = j\omega L + R$, $n = 2, \ldots N_{\max} - 1$. The average position of each impedance is $z_n^{(0)} = (n-1)\cdot\Delta L$, $n = 2, 3, \ldots N_{\max} - 1$. They are randomly distributed according to the normal law and with dispersion σ. Thus, the probability density function (PDF) for the position of the nth non-uniformity z_n is given by (Tkachenko et al., 2013b)

$$p(z_n) = (\sqrt{2\pi}\sigma)^{-1} \cdot \exp\left(-(z_n - z_n^{(0)})^2/2\sigma^2\right) \tag{14}$$

For comparison with the case of free space we consider a transmission line with the parameters: $L_{\text{TL}} = h_{\text{RES}}$, $h_{\text{TL}} = x_{0\text{RES}}$. The line is connected with matched impedances calculated by the usual way (see Fig. 2).

In the following we investigate the value $\widetilde{D} = 2Z_C \cdot J(h)/U_0$ which, for the case of a transmission line in free space, coincides with the transmission coefficient for current waves up to an obvious phase factor. We choose $N_{\max} = 6$, (i.e., the number of internal scatterers are not too large) and $N_{\text{sc}} := N_{\max} - 2 = 4$. This allows us already to show the main characteristic features of the stochastic system. The values of inductance and resistance for each impedance are: $L = 0.1\,\mu\text{H}$, $R = 50\,\Omega$. We took $N_{\text{f}} = 3000$ frequency points in the frequency band up to 3 GHz, and for each frequency point $N_{\text{st}} = 300$ statistical events.

Since the lumped impedance is mainly inductive, the scattering on it amplifies with increasing frequency. To change "the value of stochastization" of the line we changed the dispersion σ. The results for the square of the absolute value of the averaged transmission coefficient and the averaged square of the absolute value of the transmission coefficient are presented in Figs. 3 and 4 for the case of weak and strong stochastization.

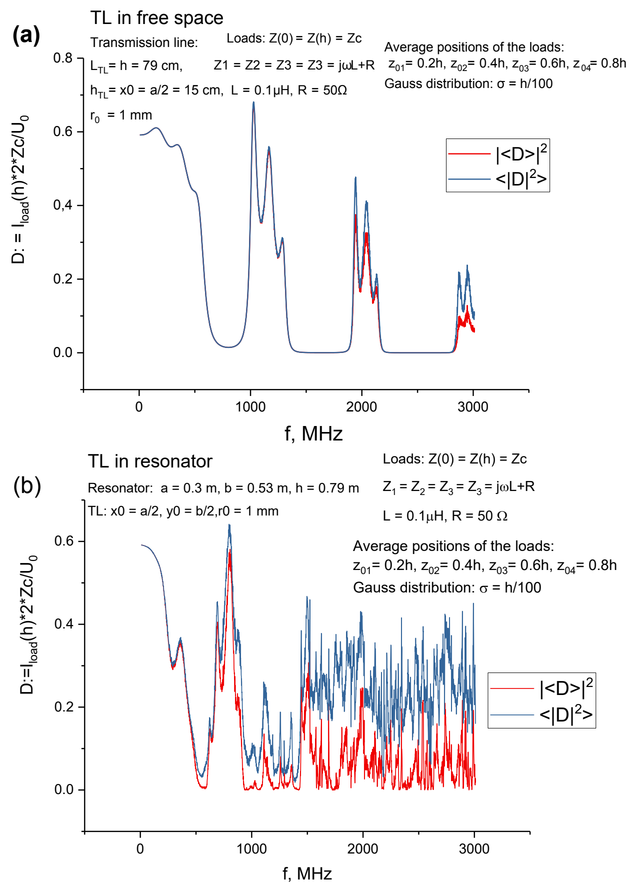

Figure 3. Statistical moments of the transmission coefficient for different parameters of the scatterers distribution. Comparisonof results for a TL in free space **(a)** and in a resonator **(b)**. Case of weak scattering: $\sigma = h/(N-1)/20 = h/100$.

For weak stochastization (see Fig. 3a) the average value of the transmission coefficient for the transmission line in free space looks practically like a transmission coefficient for the deterministic line with finite number of equidistant lumped loads (Tkachenko et al., 2013b). One observes allowed and forbidden (gap) frequency bands which are caused by the interference effects for current waves. However, for the TL in resonator, the stochastization is essentially larger $\langle |D|^2 \rangle >> |\langle D \rangle|^2$ (see Fig. 3b).

For strong stochastization for the TL in free space, only the two lowest allowed bands are observed. For higher frequencies, when the value of the impedance increases, the current wave cannot propagate through the chain (in average) (see Fig. 4a). In other words, one observes the effect of stochastic localization, where the current wave is not able to go through the stochastic line. On the contrary, for the TL in resonator the signal can propagate from the left terminal to the right

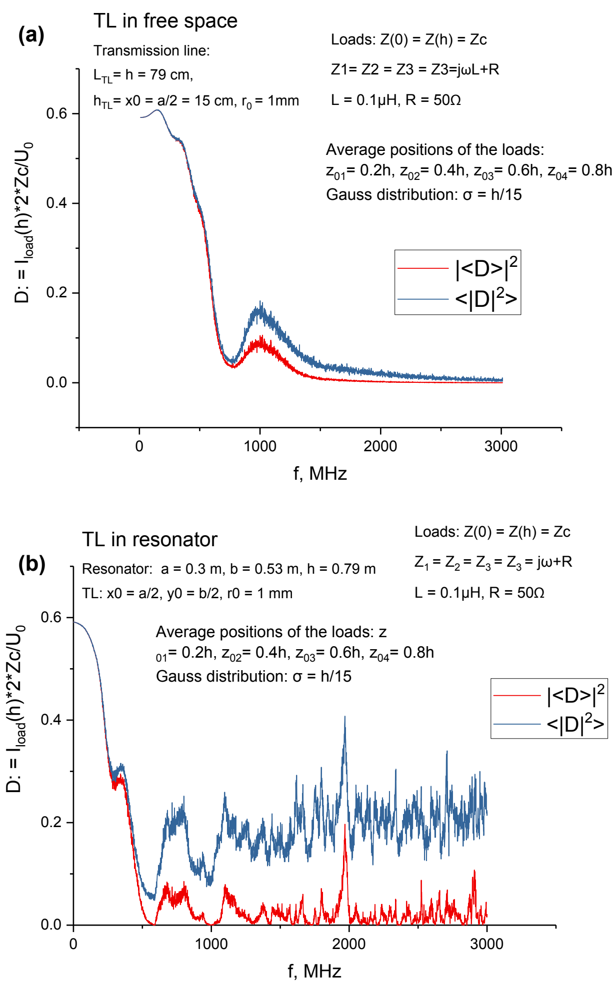

Figure 4. Statistical moments of the transmission coefficient for different parameters of the scatterers distribution. Comparison of the results for a TL in free space **(a)** and in a resonator **(b)**. Case of strong scattering: $\sigma = h/(N-1)/3 = h/15$.

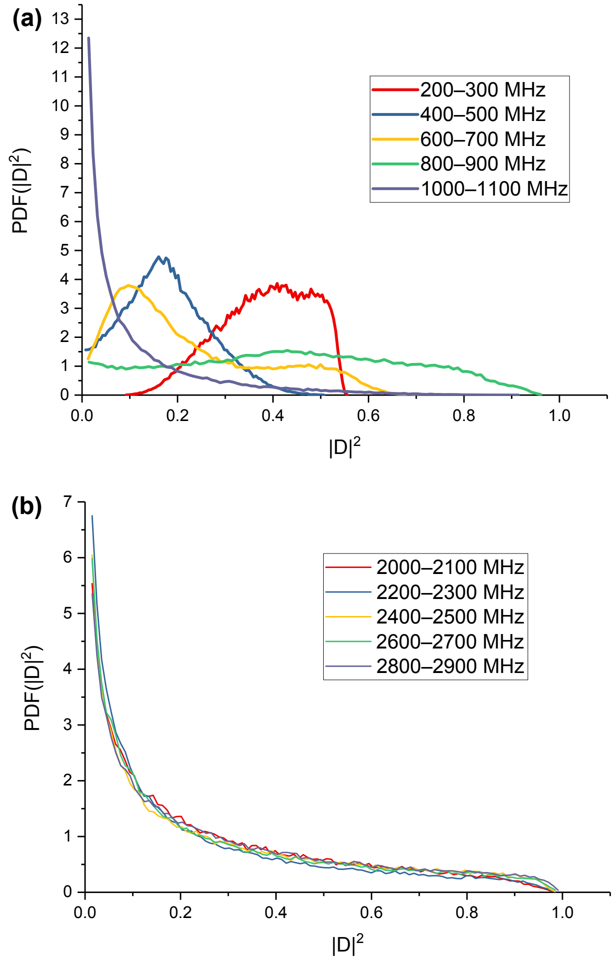

Figure 5. PDF of $|D|^2$ for different frequency intervals. **(a)** PDF for frequency intervals 200–1100 MHz. **(b)** PDF for frequency intervals 2000–2900 MHz.

one (see Fig. 4b). Roughly speaking, this can be explained as a direct action of the EM field which is re-reflecting from the walls of the resonator.

4 PDF of the transmission coefficient for a transmission line with randomly located non-uniformities inside a resonator

In this Section we investigate the probability distribution function (PDF) for the absolute value of the square of the transmission coefficient for a stochastically loaded transmission line in a resonator. We use the results of the numerical simulation for the model of stochastic variation of the positions of the lumped impedances (see above). To smooth the influence of the cavity resonances on the PDF-function and to increase the number of statistical events we consider frequency intervals of 100 MHz to define each PDF. We considered stochastical distributions with $\sigma = h/(N-1)/20 = h/100$, when the stochastic phenomena were observed (see Fig. 3b).

To find the PDF we use the usual histogram method with division of the interval from minimum to maximum of the explored values of $N_{\text{gist}} = 100$ subintervals (bins), and calculating the number of statistical events falling into each of them, followed by a further normalization of the curve. The results are presented in Fig. 5 for different frequency intervals.

Striking in the curves of Fig. 5 (Vick, 2014) are their large "humps" on the right or in the central part for the case of weak scattering (curves 200–300 and 400–500 MHz in Fig. 5a), and more or less horizontal curves for the intermediate case, and damped curves, which have their maximum near zero, for the case of strong scattering (all curves on Fig. 5b). However, unlike in the case of free space, the PDF-curve for strong scattering does not attenuate exponentially to zero, but retains a finite value. This means that, on contrary

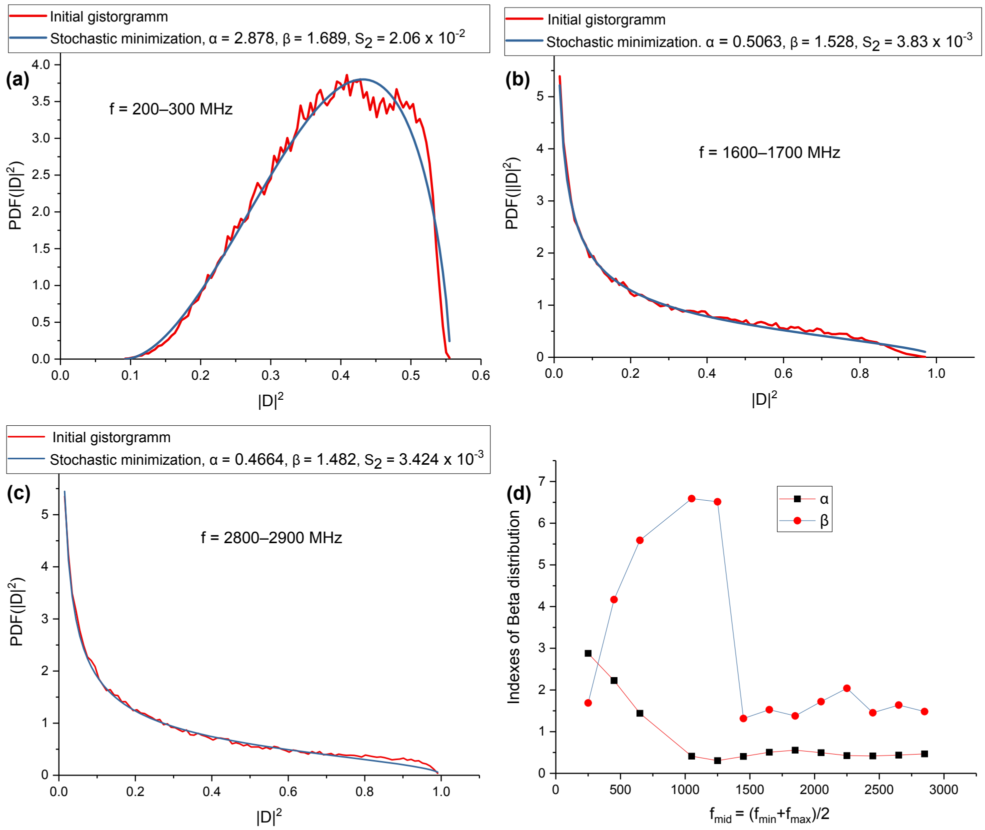

Figure 6. Beta approximation of the PDF of $|D|^2$. **(d)** Indices of the Beta approximation for the PDF of $|D|$.

to the TL in free space, the PDF-dependencies show the possibility of propagation of current waves along the stochastic line inside the resonator.

Now we can obtain an analytical approximation of the PDF-curves. The square of the absolute normalized value of the transmission coefficient $y_1 := |D|^2/|D|^2_{\max}$ is a stochastic value[1], which is defined on the restricted interval $0 < y_1 < 1$.

Then, according to the principle of maximum entropy, the PDF of y_1 is a Beta-distribution (Park and Bera, 2009):

$$f_\beta(\alpha,\beta,y_1) = y_1^{\alpha-1}(1-y_1)^{\beta-1}B^{-1}(\alpha,\beta) \quad \text{where} \quad B(\alpha,\beta)$$
is the Euler Beta-function, and parameters $\alpha > 0, \beta > 0$. (15)

The approximation parameters α and β are found by the Monte-Carlo minimization of the integral functional of the square of the absolute value of deviation S_2, where the y_{1n} and $f(y_{1n})$ are arguments and function of PDF-distribution obtained from the simulation above. The function $f_\beta(\alpha,\beta,y_{1n})$ is the beta-distribution (Eq. 15), where the indices α and β are chosen randomly from some initially guessed interval (we use 10^6 statistical events).

$$S_2 = \sum_{n=1}^{N_{\text{gist}}} \left| f_\beta(\alpha,\beta,y_{1n}) - f(y_{1n}) \right|^2 \Delta y \qquad (16)$$

For more accuracy the procedure can be repeated for narrower initial parameter intervals. The results for several frequency intervals are presented in Fig. 6a–c. In Fig. 6d we present the indices of the Beta approximation for PDF distribution of the square of the absolute value of the transmission coefficients through the chain of the stochastically arranged impedances. Analyzing this curve, one can observe a change in the nature of the scattering of the current wave at about 1300 MHz (a sharp change of the alpha-index). In our opinion, this corresponds to a transition from a regime of weak scattering to the regime of strong scattering.

5 Conclusions

In this paper, we investigated the propagation of current waves through a transmission line with a randomly located chain of lumped impedances inside a rectangular resonator using the method of symmetrical lines in the resonator, developed earlier. It was shown, that unlike the case of TL in

[1] Of course, to speak correctly about the PDF one has to have a case of strong stochastization, i.e., the equality $< D > \approx 0$ has to be satisfied.

free space, where the current wave cannot propagate through a random chain of impedances due to the effect of dynamical localization (Klyatskin, 2008), or one-dimensional Anderson localization (Lifshitz et al., 1988). In the resonator, however, such penetration is possible due to reflection of the signal from the walls of the resonator. It can be interpreted as destruction of the one-dimensional Anderson localization by additional external resonances.

The stochastical properties of the absolute value of the propagation coefficient was studied. It was shown that its PDF can be approximated by a binomial distribution with good accuracy.

The results obtained, after additional examinations, can be used to study cabling in cars, computer cases and aircraft fuselages. Namely, distortion of the signals during the penetration through internal wires can be considered and the influence of the wiring currents on the fields can be calculated. To do this, one has to solve coupling problem for multiconductor case, consider wiring in the cylindrical resonators, and to solve scattering problem. The first two problems can be solved by method of symmetrical wire in resonator (Tkachenko et al., 2018, 2014b). The main challenge here is the definition of load parameters of the examined objects, such as the number of impedances, the stochastic distributions, etc. To solve this problem, some efforts were made in (Vick, 2018) where a wire with stochastic geometry in the resonator was considered.

Data availability. The data used to support the finding of this study are available from the corresponding author upon request.

Competing interests. The authors declare that they have no conflict of interest.

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