

Determination of Harmonic-Transfer-Matrices by Simulation

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Abstract. In this paper a method to obtain harmonic transfer matrices (HTM) from simulated or measured values (signature & signature response) is presented. These matrices subserve a description of complex systems (e.g. RF front-ends) with real properties, which can't be specified by simple analytic expressions. They afford to give statements about a systems parameter. That's why HTM are suitable for Built-In-Self-Test (BIST) and opens the option for Built-In-Self-Correction (BISC) for critical function-blocks.

1 Introduction

For specification compliant function of modern communication systems a permanent check of operating parameters is necessary. Within higher integration density it is getting more and more difficult to test single components and readjust them. The tested components have to maintain well-defined parameters else a whole system can be inoperable. The common separate testing of single devices is elaborate and expensive because there are high requirements in hardware and time. Whereas Radio-Frequency Built-In-Self-Test (RF-BIST) also offers a concept for testing whole RF front-ends. On functional devices integrated test-circuits realise an automatic function-test on production and at starting the device. Signatures and their responses provide information about state and behaviour of system. For characterisation the systems properties harmonic transfer matrices (HTM) can be used. Such matrices describe transmission behaviour of circuits between frequencies of input signal and frequencies of output signal. It's possible to depict effects like intermodulation and frequency conversion appearing in nonlinear systems. Intension is to make conclusions from distinctive matrix-structures to potential readjustable operation parameters. Fixing incorrect circuit-parameters from HTM facilitate a targeted correction. Built-In-Self-Correction (BISC) as next step is applicable in order to latter point.

In this paper a method is presented to ascertain HTM from input and output signals in conformity with measured signals. The creation of HTM depends on special conditions.

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This has an outstanding meaning concerning nonlinear systems and their functional characterisation, because there is coherence between HTM and circuit structure. Matrices for LTI- and linear periodically time variant systems (LPTV) can be calculated easily. It is examined, how to get HTM from input and output signals. The verification of developed algorithms is carried out by simulation on transistor level.

The text is structured as follows. Section 2 is responsive to the basics of HTM. Subsequently in Sect. 3 essential models for block description are presented. In Sect. 4 the block description is emphasised to nonlinear systems and the calculation of HTM is explained. Simulation results for a RC-low-pass filter and a real mixer are pictured in Sect. 5. Conclusions are made in Sect. 6.

2 Harmonic Transfer Matrices (HTM)

LTI systems are described by transfer functions. For linear periodically time variant systems (LPTV) a simple modelling with transfer functions isn't feasible. The transfer properties of LPTV systems can be expressed by harmonic transfer matrices (HTM). For last-mentioned systems the HTM characterises small signal behaviour while large signal operation point changes periodically in time. Examples therefore are mixer and phase locked loop (PLL). The derivation of HTM for LPTV systems is given in Vanassche et al. (2002, 2003).

The relation between input and output signal is given by:

$$y(t) = \int_{-\infty}^{\infty} h(t, r)x(r)dr. \quad (1)$$

The systems pulse response $h(t, r)$ is supposed to be known and periodically in time. The input signal in time domain is $x(r)$.

Converting Eq. (1) like in Vanassche et al. (2002) and Vanassche et al. (2003) reaches to equation:

$$Y(s + ls_0) = \sum_{k=-\infty}^{\infty} H_{l-k}(s + ks_0)X(s + ks_0). \quad (2)$$

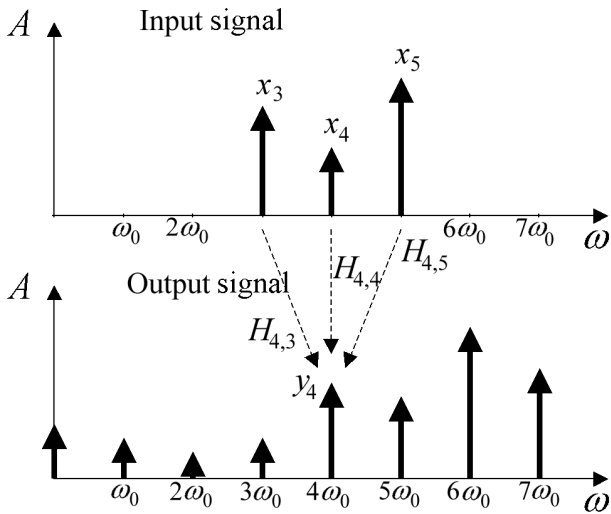


Fig. 1. Influence of single spectral components from input to output.

How to see easily the vector on right side in Eq. (2) can be written as a product from matrix and a vector X .

$$\begin{bmatrix} \vdots \\ Y(s - s_0) \\ Y(s) \\ Y(s + s_0) \\ \vdots \end{bmatrix} = \begin{bmatrix} \ddots & \vdots & \vdots & \vdots & \\ \cdots & H_0(s - s_0) & H_1(s) & H_2(s + s_0) & \cdots \\ \cdots & H_{-1}(s - s_0) & H_0(s) & H_1(s + s_0) & \cdots \\ \cdots & H_{-2}(s - s_0) & H_{-1}(s) & H_0(s + s_0) & \cdots \\ \vdots & \vdots & \vdots & \ddots & \end{bmatrix} \cdot \begin{bmatrix} \vdots \\ X(s - s_0) \\ X(s) \\ X(s + s_0) \\ \vdots \end{bmatrix} \quad (3)$$

In Eq. (3) the assignment of input vector X and output vector Y by HTM is shown. There are infinite numbers of rows and columns in HTM. The elements contain Fourier-coefficients of the periodic pulse response. If there are only a finite number of elements in the HTM, there is an error according to the remainder term of the Fourier series.

X and Y contain the Fourier-coefficients of input signal $x(t)$ and output signal $y(t)$. Generally the HTM can be written as (Vanassche et al., 2002):

$$\mathbf{H} = \begin{bmatrix} \ddots & \vdots & \vdots & \vdots & \\ \cdots & H_{-1,-1} & H_{-1,0} & H_{-1,1} & \cdots \\ \cdots & H_{0,-1} & H_{0,0} & H_{0,1} & \cdots \\ \cdots & H_{1,-1} & H_{1,0} & H_{1,1} & \cdots \\ \vdots & \vdots & \vdots & \ddots & \end{bmatrix} \quad (4)$$

with $H_{l,k} = H_{l-k}(s + ks_0)$. Every row of HTM specifies the effect of input signal $x(t)$ on every spectral component of output signal $y(t)$ (Fig. 1). The matrix-elements indicate weighted influence of a spectral component from input signal on a spectral component of output signal.

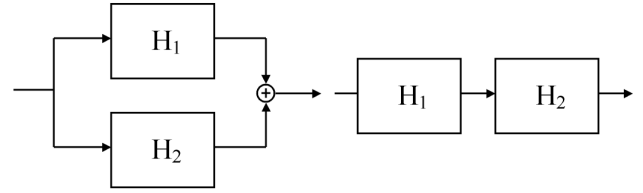


Fig. 2. Parallel connection (left) and series connection (right).

Simultaneously there can occur phase shift from input to the output of a system. Hence matrix and vector elements are complex numerical values.

3 Essential models

Ideal linear amplifier

An ideal linear amplifier actualises a multiplication of input signal with a constant. In this case it doesn't matter if the input signal is a multi-tone signal. The amplification is frequency independent and there is no frequency conversion. The main-diagonal elements contain a constant amplification factor a , and all other elements are zero.

$$y(t) = a \cdot x(t) \rightarrow \mathbf{Y} = \mathbf{H}_A \cdot \mathbf{X}; \quad H_{A;n,n} = a : \forall n \quad (5)$$

The parameter a is the amplification. If it is a complex value, it additionally realises a phase shift.

Ideal mixer

An ideal mixer causes only frequency conversion. Every spectral component gets the same frequency offset.

$$y(t) = a \cdot e^{j\omega t} x(t) \rightarrow \mathbf{Y} = \mathbf{H}_M \cdot \mathbf{X} \quad (6)$$

$$H_{M;m,n} = a : \forall n = m + offset$$

Only the elements in a secondary diagonal consists of the constant factor a . Even the main diagonal is zero. If $a=1$, pure frequency conversion will be effected. Amplitude and phase won't be changed. If $a \neq 1$ there will an effect like a combination from mixer with $a=1$ and amplifier with $a \neq 1$.

LTI-systems

Ideal linear amplifier, attenuator and linear passive filter are LTI-systems.

These systems apply:

$$\begin{aligned} H_{l,k}(s) &= H(s + ks_0) = G(s + ks_0) \quad \text{for } k = l \\ H_{l,k}(s) &= 0 \quad \text{for } k \neq l \end{aligned} \quad (7)$$

While $G(s)$ is the transfer function of the LTI-system.

Real Devices

Real devices can be modelled by interconnecting ideal devices. That will be done with the above introduced ideal models shown in Fig. 2. The resulting HTM is computed by interconnected ideal subsystems and their HTM.

Calculation rules for this purpose are shown in Lupea et al. (2003) and Vanassche et al. (2003). It applies for block level:

Parallel connection : $\mathbf{H}_{ges} = \mathbf{H}_1 + \mathbf{H}_2$ (8)

Series connection : $\mathbf{H}_{ges} = \mathbf{H}_2 \cdot \mathbf{H}_1$ (9)

Furthermore the calculation rules for matrices are used for HTM as shown in Vanassche et al. (2002).

More details for important models to describe systems with HTM are given in Lupea et al. (2003).

4 Synthesis of HTM

4.1 Extending HTM for nonlinear systems

HTM like in Sect. 2 are only given for LPTV-systems, because Laplace and Fourier transformation respectively claims linearity. However the matrices are applicable to non-linear static systems. The frequency conversion effect can be used as approach. The conversion is modelled by mixers' HTM (q.v. also Lupea et al., 2003). Additional characteristics are expressible by amplifiers and filters. It's important to remark that HTM for nonlinear cases are only valid for a steady operation point and input signals periodic in time. Otherwise there is no HTM expressed by mixer models assignable. Changing operation point there is also a variation in frequency conversion and intermodulation with regard to the relation of input and output signals. The periodicity of input signal is precondition to specify a HTM. There is no way to generate a Fourier series to a non-periodic signal. Hence there is also no vector notation for this signal. Thus HTM for nonlinear systems depends on operation point and input signal. For example a nonlinear amplifier has a characteristic:

$$y(t) = a_1 x(t) + a_3 [x(t)]^3. \tag{10}$$

The amplifier can be modelled as shown in Fig. 3. The nonlinearity $[x(t)]^3$ is realised with two ideal mixers M_1 and M_2 . At both inputs of mixer M_1 the input signal $x(t)$ is injected. The output signal $[x(t)]^2$ is connected with one input of mixer M_2 . The other input is fed with $x(t)$. So M_2 emits $[x(t)]^3$. The ideal amplifiers A_1 and A_3 are linear. With A_1, A_3, M_1, M_2 as HTM as described in Sect. 3, then is:

$$Y = [A_1 + A_3 \cdot M_2 \cdot M_1] \cdot X. \tag{11}$$

4.2 The characteristic vector

A "characteristic" vector of HTM, which don't describe filter functions, is constituted. That means this vector is just significant to amplifier and mixer matrices and their sums and products, because only such matrices are Toeplitz matrices. Toeplitz matrices M have following property:

$$M \in C^{m \times n}; \forall i, j : M_{i,j} = M_{i+1,j+1} \tag{12}$$

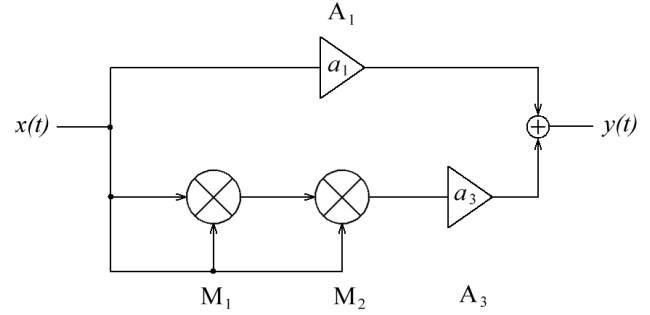


Fig. 3. Block-model of a nonlinear amplifier.

and is a basic characteristic of HTM of mixers and nonlinear systems without filter performance. The HTMs row with index zero is declared as the characteristic vector V_0 :

$$H = \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \dots & b & c & \dots & \dots \\ \dots & a & b & c & \dots \\ \dots & \dots & a & b & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} = \begin{bmatrix} \vdots \\ V_{-1} \\ V_0 \\ V_1 \\ \vdots \end{bmatrix}. \tag{13}$$

So there emerges a demand for completeness to this vector. At least there should be minor error if elements are missing. The amplifier example above implies that the mixer matrices M_1 and M_2 are identical and merely contain the input signal x .

4.3 Multiplication of HTM

It is necessary to regard that the matrices M_1 and M_2 have an infinite number of rows and columns. If the characteristic vector is bounded in length, i.e. all elements $v_i=0$ with $|i|>i_{max}$, then the matrix is allowed to have a finite number of rows and columns. For numerical processing are only matrices with finite dimension evaluable anyway. But it is not possible to calculate with quadratic matrices, because they don't include the whole characteristic vector in every row. That is why there is the need to render matrices which have to be multiplied. The left factor gets more columns and the right factor more rows, so there is the entire characteristic vector in every row.

The left factor M_2 according to the amplifier example is:

$$M_2 = \begin{bmatrix} a & b & c & d & e \\ & a & b & c & d & e \\ & & a & b & c & d & e \\ & & & a & b & c & d & e \\ & & & & a & b & c & d & e \end{bmatrix} \tag{14}$$

with the characteristic vectors:

$$V_{M1} = V_{M2} = [a \ b \ c \ d \ e]. \tag{15}$$

The multiplication of these matrices realises just a cross correlation of their two characteristic vectors at last.

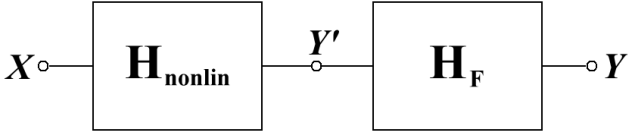


Fig. 4. Partitioning a separable system in blocks.

The product of HTM and input vector X results in vector Y . In ideal case latter contains the same Fourier coefficients as the signal $y(t)$. For additional frequency-selective behaviour the HTM has to be multiplied with a filter-HTM \mathbf{H}_F . The resulting HTM yields as follows:

$$\mathbf{H}_{\text{res}} = \mathbf{H}_F \cdot \mathbf{H}_{\text{Toeplitz}} \quad (16)$$

4.4 Vectorial representation

\mathbf{H}_{res} is not a Toeplitz matrix, because filter-HTMs don't have such a structure. Hence there is no characteristic vector presentable. Splitting \mathbf{H}_{res} as given in Eq. (16) enables to describe the whole HTM with two vectors, the characteristic vector and a filter vector. This notation is space-saving in contrast to the HTM. Latter fact is particularly clear if the matrices have more than 100×100 elements. Moreover the partition Eq. (16) is a fundamental assumption to the method to compute the HTM as presented in Sect. 4.5.

The filter vector F only consists of the main diagonal components of filter matrix \mathbf{H}_F .

$$\mathbf{H}_F = \begin{bmatrix} \ddots & & & & & \\ & G_v & & & & \\ & & G_{v+1} & & & \\ & & & G_{v+2} & & \\ & & & & \ddots & \end{bmatrix} \rightarrow F = \begin{bmatrix} \vdots \\ G_v \\ G_{v+1} \\ G_{v+2} \\ \vdots \end{bmatrix} \quad (17)$$

4.5 Computation of HTM

There are following restrictions for calculation of HTM:

- The input signal $x(t)$ is an OFDM-signal. That means it is available as:

$$x(t) = \sum_{i=1}^n x_i \cos(\omega_i t + \varphi_i), \quad x_i = 0 \quad \text{with} \quad i = 0. \quad (18)$$

- The system to analyse is a linear or a nonlinear separable system. Latter is a system which filter characteristic is separable from nonlinear characteristic, so both characteristics can be expressed by two HTM as shown in Fig. 4.

It yields:

$$Y = \mathbf{H}_F \cdot \mathbf{H}_{\text{nonlin}} \cdot X. \quad (19)$$

A measurement of large signals $x(t)$ and $y(t)$ is accomplished and vectors X and Y determined. From a small signal

measurement between input and output results the filter matrix. So nonlinear effects are negligible. With compliance of restrictions given above, the HTM describing nonlinearities contains redundancies which reduces the number of variables in the arrangible system of equations.

Hence it is:

$$Y' = \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} H_2^* & H_1^* & H_0 & H_1 & H_2 \\ 0 & H_2^* & H_1^* & H_0 & H_1 \\ 0 & 0 & H_2^* & H_1^* & H_0 \\ 0 & 0 & 0 & H_2^* & H_1^* \\ 0 & 0 & 0 & 0 & H_2^* \end{bmatrix} \cdot \begin{bmatrix} x_{-2} \\ x_{-1} \\ x_0 \\ x_1 \\ x_2 \end{bmatrix} \quad (20)$$

The system of equations can be solved by splitting the single elements in real part and imaginary part. For Y' will be written Y further.

$$Y_{\text{re}} + Y_{\text{im}} = \mathbf{H}_{\text{re}} \cdot X_{\text{re}} - \mathbf{H}_{\text{im}} \cdot X_{\text{im}} + j(\mathbf{H}_{\text{re}} \cdot X_{\text{im}} + \mathbf{H}_{\text{im}} \cdot X_{\text{re}}) \quad (21)$$

So there yields a new system of equations:

$$\begin{bmatrix} Y_{\text{re}} \\ Y_{\text{im}} \end{bmatrix} = \begin{bmatrix} \mathbf{H}_{\text{re}} & -\mathbf{H}_{\text{im}} \\ \mathbf{H}_{\text{im}} & \mathbf{H}_{\text{re}} \end{bmatrix} \cdot \begin{bmatrix} X_{\text{re}} \\ X_{\text{im}} \end{bmatrix} \quad (22)$$

By re-sorting the matrix elements follows:

$$\begin{bmatrix} Y_{\text{re}} \\ Y_{\text{im}} \end{bmatrix} = \mathbf{X}_M \cdot \begin{bmatrix} V_{\mathbf{H}_{\text{re}}} \\ V_{\mathbf{H}_{\text{im}}} \end{bmatrix} \quad (23)$$

where \mathbf{X}_M consists of elements of X_{r} and X_{i} and

$$V_{\mathbf{H}_r} = [Re(H_0) \quad Re(H_1) \quad \dots \quad Re(H_n)]^T.$$

The imaginary part of $V_{\mathbf{H}}$ yields in equivalent way to the real part. Equation (23) specifies a over-determined system of equations. In most cases there is no solution. To solve this problem least mean square algorithms (LMS) are used. The wanted variables can be calculated as shown in Eqs. (24) and (25):

The system of equation is:

$$\mathbf{A} \cdot \mathbf{x} = \mathbf{b}. \quad (24)$$

The LMS-condition is:

$$\sum_{i=1}^m r_i^2 = \mathbf{r}^T \cdot \mathbf{r} = (\mathbf{A}\mathbf{x} - \mathbf{b})^T \cdot (\mathbf{A}\mathbf{x} - \mathbf{b}) = \min \quad (25)$$

with $\mathbf{r} = (\mathbf{A}\mathbf{x} - \mathbf{b})$, $\mathbf{r} \neq 0$ and

$$\mathbf{A} = (a_{i,j}), \quad i = 1 \dots m, \quad j = 1 \dots n, \quad m > n.$$

5 Simulation results

To verify operability of developed algorithm the input and output vectors have been created with a network simulator. A DUT has been simulated with a OFDM-input signal $x(t)$, modulated in amplitude, and the output signal $y(t)$ has been detected. The spectra of the signals yield in input and output vectors. The amplitude modulation in small signal range let determine the filter matrix. Fig. 5 shows the characteristic

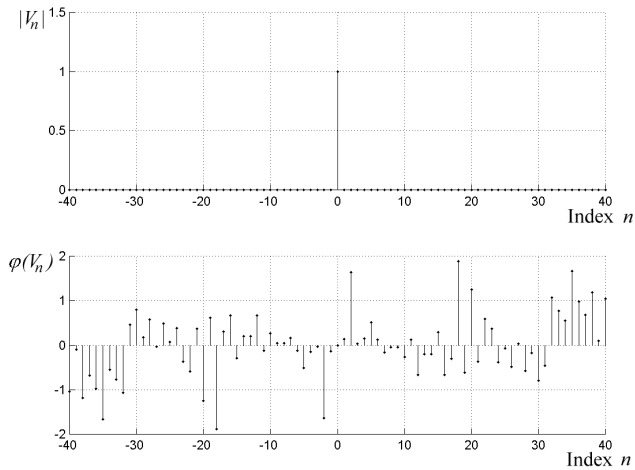


Fig. 5. Characteristic vector of RC-low-pass filter.

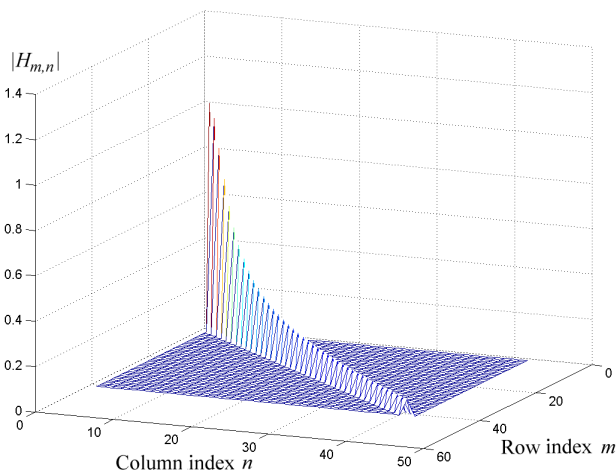


Fig. 6. HTM-Section of a RC-low-pass filter.

vector of a simple RC-low-pass filter. With the method described above this vector has been correctly calculated. The element with index 0 got the value 1, all other are ideally 0. As a result of inaccuracies in computation there are minimal tolerances. But in phase are huge deviations when magnitude is very small. It should be noted that the phase of complex values with a zero magnitude is undetermined. The characteristic vector creates an identity matrix for the nonlinear behaviour, because the filter matrix already describes the whole behaviour of this LTI-system.

Figure 6 shows the magnitude of a section of the HTM. The resulting HTM includes the gain response following the main diagonal.

A tested mixer has the gain response and the phase response pictured in Fig. 7. The characteristic vector is shown in Fig. 8. Latter let recognise the structure for mixer and nonlinearities. The typical amplitudes, effected by local oscillator, are explicitly visible at index ± 10 . That corresponds $f_{LO}=1$ GHz. The linear amplifying component is located at

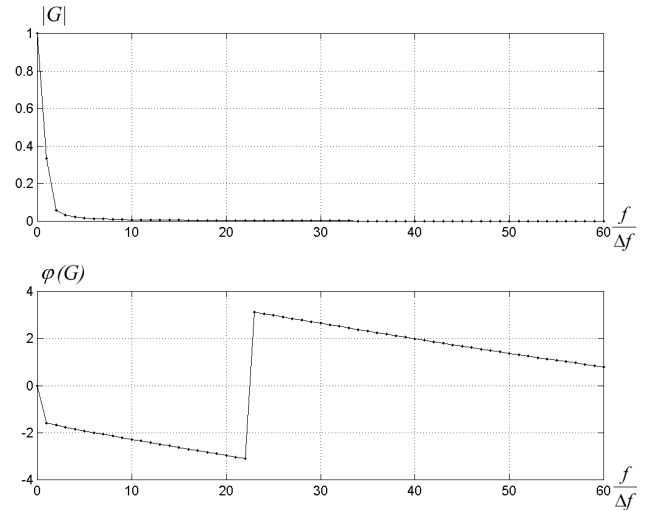


Fig. 7. Amplitude and phase response of a mixer.

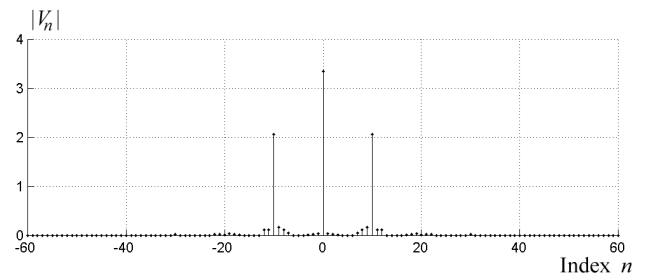


Fig. 8. Characteristic vector of a mixer.

index 0 (0 Hz). The small involutions round the oscillator and linear peaks originate from nonlinear distortions caused by intermodulation.

The mixer has isolating capacitors at signal input and output. The small signal for analysing the filter characteristic has to pass these capacitors. So it is possible that no valid voltages could be measured, because the constant component is filtered out. The filter function has to be questioned for such cases. In an iterative network of filters previous filter can mask the test-signal for subsequent devices. The amplitudes of small signal can become less than it would be necessary for computation.

6 Parameter-extraction from HTM

With structure of HTM and characteristic vector respectively assertions about a system can be made. So induces the occupancy of secondary diagonal elements the existence of mixer behaviour. But it is not possible to maintain offhand the frequency conversion is generated by a mixer or by intermodulation, because nonlinearities are modelled by mixer matrices. To make states concerning BISC, the properties of DUT has to be known in such way, that a comparison between

“determined” HTM and a nominal HTM can be made. Then parameter variation and correction values can be calculated.

7 Conclusions

In this paper a method to determine harmonic transfer matrices (HTM) by simulation has been presented. Starting point was the assumption for description of HTM of linear periodic time variant systems (LPTV). The concept of HTM has been extended on nonlinear static systems. A synthesis from ideal building blocks to HTM, specifying real systems, utilising fundamental properties of HTM has been arranged. A way to calculate HTM for separable systems was shown. The HTM can be presented in short view with two vectors. With simulations the synthesis has been verified, whereas the applicability for BIST and BISC became obvious.

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